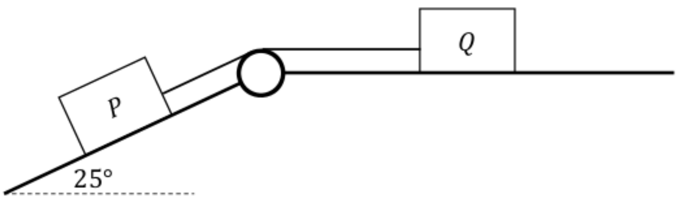
Connected Particles Exam Question Solutions

Note These exam questions are answered in reverse chronological order as they appear in exam papers; 2023 paper, Sample paper, 2022 (deferred), 2022, and so on back to 2015. They are answered in the style described in my notes. Only questions from the old syllabus relevant to the new syllabus are included, including those that are not Q4 but have enough of a Connected Particles component to be considered worthwhile when revising the topic.

Question — 2023 Q5 (a).

Question 5

(a) Block P (of mass 6.3 kg) and block Q (of mass 2.5 kg) are held at rest on a rough surface. They are connected by a light inextensible string which passes over a smooth fixed pulley. Block Q lies on the horizontal part of the surface and block P lies on the part of the surface that is inclined at 25° to the horizontal, as shown in the diagram.



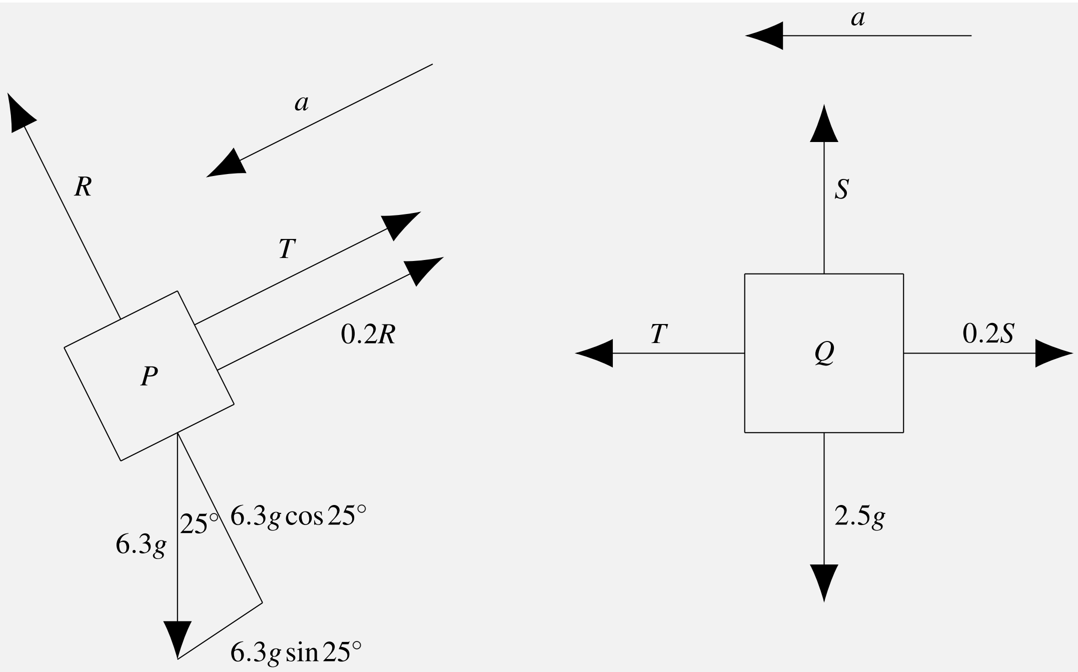
The coefficient of friction between each block and the surface is 0.2.

The blocks begin to move when they are released.

(i) Show, on separate diagrams, the forces acting on the blocks while they are moving.

(ii) Calculate the acceleration of the blocks.

(i)



(ii) Our equations are

$$\begin{aligned}6.3g\sin 25^\circ - T - 0.2R &= 6.3a,\\ R &= 6.3g\cos 25^\circ\\ T - 0.2S &= 2.5a\\ S &= 2.5g.\end{aligned}$$

Substituting R and S into the first and third equation and adding them gives them

$$\begin{aligned}6.3g\sin 25^\circ - T - 1.26g\cos 25^\circ &= 6.3a\\ (+) \qquad T - 0.5g &= 2.5a\\ \hline 6.3g\sin 25^\circ - 1.26g\cos 25^\circ - 0.5g &= 8.8a\\ \Rightarrow 1.14 \text{ m/s}^2 &= a.\end{aligned}$$

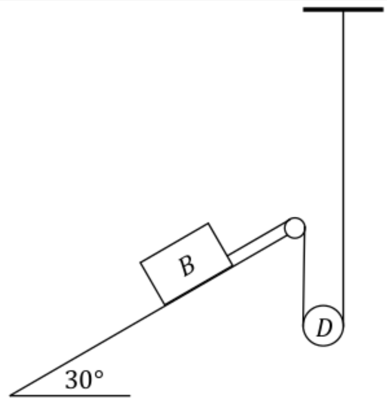
Question — Sample Q7 (b).

(b) A small smooth moveable disk D, of mass 0.2 kg, rests on a light inextensible string. One end of the string is connected to block B, of mass 4 kg, which rests on a rough plane inclined at 30° to the horizontal.
The other end of the string is connected vertically to a fixed point.

The coefficient of friction between block B and the inclined plane is $\frac{1}{10}$.

When the system is released from rest, D moves upwards with acceleration a.

The tension in the string is T.

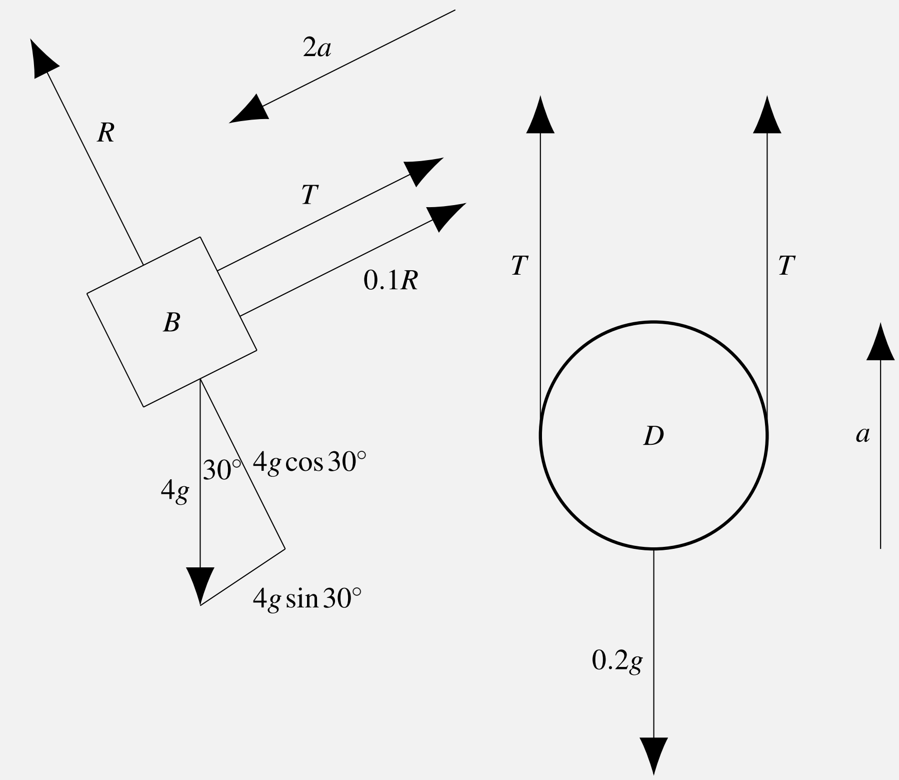


(i) Show, on separate diagrams, the forces acting on block B and disk D while they are moving.

(ii) Explain why the acceleration of B is $2a$.

(iii) Calculate a and T.

(i)



(ii) If D moves up x m, then there are $2x$ m of rope fed to B. Therefore it moves down $2x$ m. As they both start from rest, this implies that the speed of B is twice the speed of D, as is its acceleration.

(iii) Our equations are

$$R = 4g\cos 30^\circ$$
$$4g\sin 30^\circ - T - 0.1R = 4(2a)$$
$$2T - 0.2g = 0.2a.$$

Substituting R into the second equation and calculating the trigonometric values,

$$2g - T - 0.2g\sqrt{3} = 8a$$
$$2T - 0.2g = 0.2a.$$

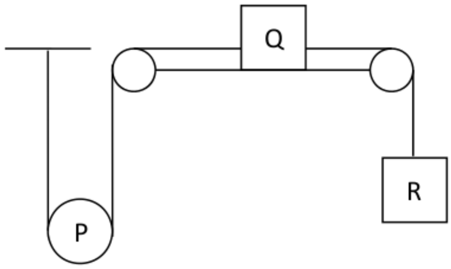
Multiplying the first equation by 2 and adding them,

$$\begin{aligned} & 4g - 2T - 0.4g\sqrt{3} = 16a \\ (+) \quad & 2T - 0.2g = 0.2a \\ \hline & 3.8g - 0.4g\sqrt{3} = 16.2a \\ & \Rightarrow \frac{3.8 - 0.4\sqrt{3}}{16.2}g = a \\ & \Rightarrow 1.88 \text{ m/s}^2 = a \\ & \Rightarrow 2T - 0.2g = 0.2(1.88) \\ & \Rightarrow 2T = 2.34 \\ & \Rightarrow T = 1.17 \text{ N.} \end{aligned}$$

Question — 2022 (Deferred) Q4 (a).

(a) A taut light inelastic string is fixed at one end and passes under a moveable pulley, P, of mass 4 kg which hangs vertically. The other end of the string is attached to Q, a mass of 4 kg which lies on a rough horizontal surface.

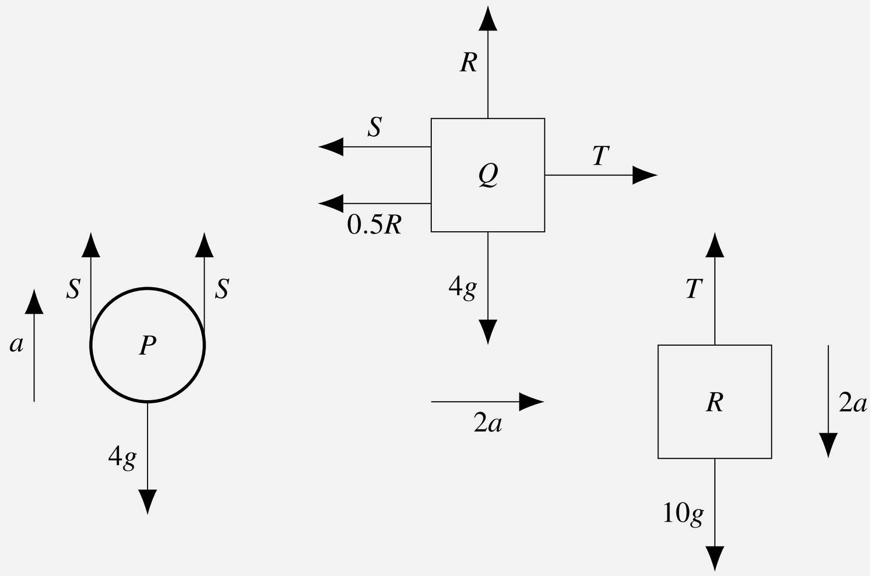
A second inelastic string connects Q to R, a mass of 10 kg which hangs vertically.



The fixed pulleys are smooth and light and the coefficient of friction between Q and the surface is $\frac{1}{2}$.

The system is released from rest.

Find the accelerations of P, Q and R in terms of g.



It's a leap to say that P goes up and R goes down, and because of friction in Q it matters if we're wrong, but I'm guessing they do because R is much heavier. Our equations are

$$\begin{aligned} 2S - 4g &= 4a \\ R &= 4g \\ T - S - 0.5R &= 4(2a) \\ 10g - T &= 10(2a) \end{aligned}$$

Substituting R into the third equation, dividing the first equation by 2 and adding the three new remaining new equations

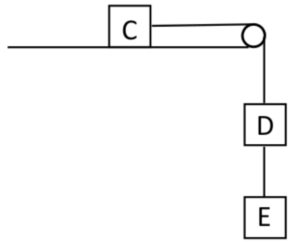
$$
\begin{aligned}
S - 2g &= 2a \\
T - S - 2g &= 8a \\
(+) \qquad 10g - T &= 20a \\
\hline
6g &= 30a \\
\Rightarrow \frac{g}{5} &= a.
\end{aligned}
$$

The acceleration of P, Q and R are then $\frac{g}{5}$, $\frac{2g}{5}$ and $\frac{2g}{5}$ respectively.

Question — 2022 Q4 (a).

(a) A block C of mass $6m$ rests on a rough horizontal table.

It is connected by a light inextensible string which passes over a smooth fixed pulley at the edge of the table to a block D of mass $3m$. D is connected by another light inextensible string to a block E of mass $2m$, as shown in the diagram.



The coefficient of friction between C and the table is $\frac{1}{3}$.

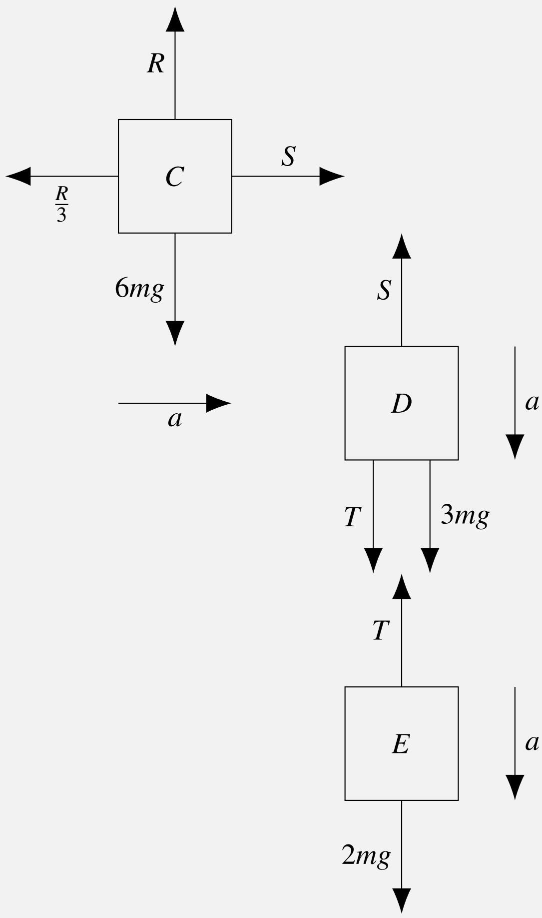
The system is released from rest.

(i) Show on separate diagrams the forces acting on each block.

(ii) Find the acceleration of C.

(iii) Find the tension in each string.

(i)



(ii) Our equations are

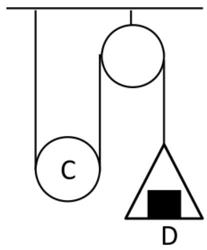
$$
\begin{aligned}
R &= 6mg \\
S - \frac{R}{3} &= 6ma \\
T + 3mg - S &= 3ma \\
2mg - T &= 2ma.
\end{aligned}
$$

Substituting R into the second equation and adding the three remaining equations we get

$$
\begin{aligned}
S - 2mg &= 6ma \\
T + 3mg - S &= 3ma \\
(+) \qquad 2mg - T &= 2ma \\
\hline
3mg &= 11ma \\
\Rightarrow \frac{3g}{11} &= a.
\end{aligned}
$$

Question — 2021 Q4 (a).

(a) The diagram shows a light inextensible string having one end fixed, passing under a smooth movable pulley C of mass *km* kg and then over a fixed smooth pulley. The other end of the string is attached to a light scale pan. A bock D of mass *m* kg is placed symmetrically on the centre of the scale pan.
The system is released from rest. The scale pan moves upwards.

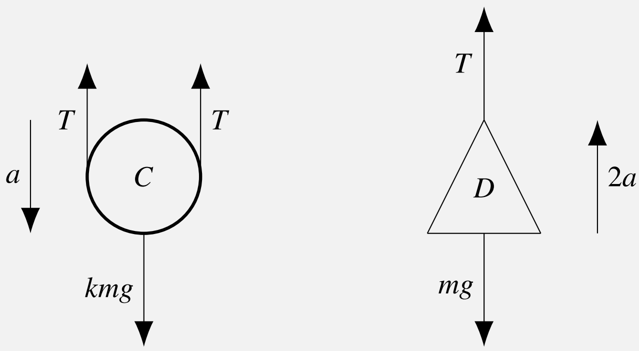


(i) Show that $k > 2$.

(ii) Find, in terms of *k* and *m*, the tension in the string.

(iii) Find, in terms of *k* and *m*, the reaction between D and the scale pan.

(i) For parts (i) and (ii) we can treat D and the scale pan as a single object.



Our equations are

$$\begin{aligned} kmg - 2T &= kma \\ T - mg &= 2ma \end{aligned}$$

Multiplying the second equation by 2 and adding them gives

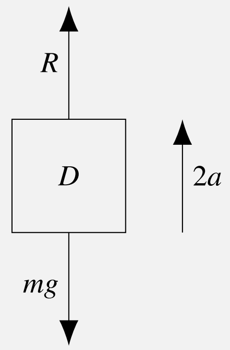
$$\begin{aligned} kmg - 2T &= kma \\ (+)\ 2T - 2mg &= 4ma \\ \hline (k-2)mg &= (4+k)ma \\ \Rightarrow \frac{k-2}{4+k}g &= a. \end{aligned}$$

As $a > 0$ we get $k > 2$ immediately.

(ii) From the second equation in (i)

$$\begin{aligned} T - mg &= 2ma \\ \Rightarrow T &= mg + 2m\frac{k-2}{4+k} \\ &= mg\left(\frac{4+k}{4+k} + \frac{2k-4}{4+k}\right) \\ &= mg\frac{3k}{4+k}. \end{aligned}$$

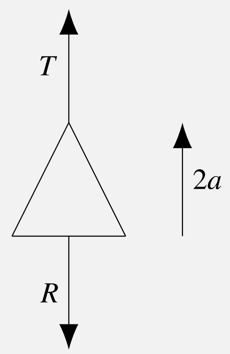
(iii) Consider the forces acting on the particle D.



As we already know a,

$$\begin{aligned} R - mg &= m(2a) \\ \Rightarrow R &= mg + m\frac{2k-4}{4+k}g \\ &= mg\left(\frac{4+k}{4+k} + \frac{2k-4}{4+k}\right) \\ &= mg\frac{3k}{4+k}. \end{aligned}$$

Alternatively we could have looked at the forces acting on the scale pan (which is light, i.e. has mass 0).

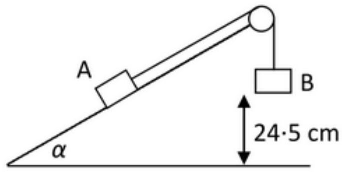


As mass is 0,

$$\begin{aligned} F &= ma \\ \Rightarrow T - R &= 0 \\ \Rightarrow R &= T \\ &= mg\frac{3k}{4+k}. \end{aligned}$$

Question — 2020 Q4.

4. **(a)** A block A of mass $10m$ on a smooth plane inclined at an angle α with the horizontal, where $\tan\alpha = \frac{3}{4}$, is connected by a light inextensible string which passes over a smooth pulley to a second block B of mass $10m$. B is 24·5 cm above an inelastic horizontal floor, as shown in the diagram.



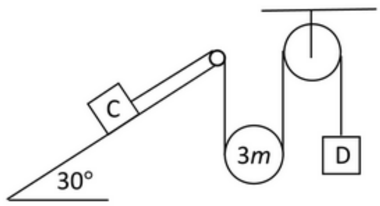
The system is released from rest.

Find

(i) the acceleration of B

(ii) the time that B remains in contact with the floor.

(b) A particle C of mass $2m$ rests on a rough plane which is inclined at 30° to the horizontal. The coefficient of friction between C and the plane is $\frac{\sqrt{3}}{21}$. A light inextensible string which passes under a smooth movable pulley of mass $3m$ connects C to a particle D of mass m, as shown in the diagram.

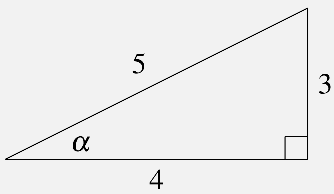


The system is released from rest. C moves up the plane.

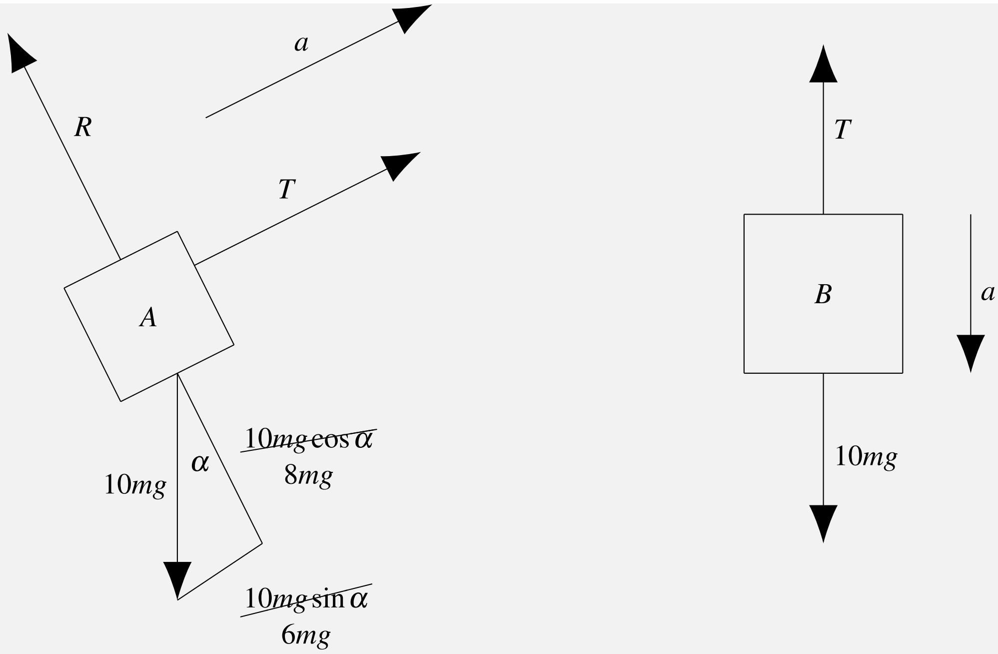
(i) Show, on separate diagrams, the forces acting on the moveable pulley and on each of the masses.

(ii) Find in terms of m the tension in the string.

(a) (i) By drawing a triangle and applying Pythagoras' Theorem, we can use $\tan\alpha = \frac{3}{4}$ to find that $\cos\alpha = \frac{4}{5}$, $\sin\alpha = \frac{3}{5}$.



Then our diagrams of forces are



Our equations are

$$R = 8mg$$
$$T - 6mg = 10ma$$
$$10mg - T = 10ma$$

Adding the second and third equation yields

$$4mg = 20ma$$
$$\Rightarrow \frac{g}{5} = a.$$

(ii) Below is a UVAST array for B's journey from its starting position to the floor.

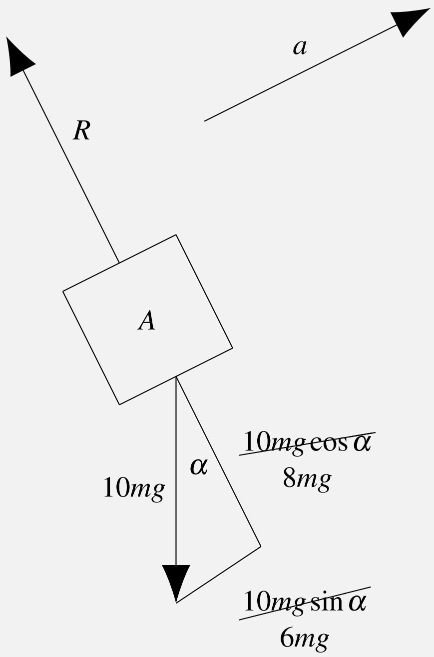
$$u = 0$$
$$v =$$
$$a = \frac{g}{5}$$
$$s = 0.245$$
$$t =$$

Then

$$v^2 = u^2 + 2as$$
$$= 0.098g$$
$$\Rightarrow v = \sqrt{0.098g}$$
$$= 0.98 \text{ m/s}.$$

After B hits the ground, A continues up the slope, but now the string is slack. A will continue up the slope until it reaches a maximum height, before sliding down the slope and making the string taut again. The question doesn't explicitly state it but presumably when the string goes taut B lifts again. This is the journey we want to concentrate on, and we need three quantities for the UVAST column for this journey.

The speed of A at the beginning of this journey is equal to the final speed of B when it hits the ground. The displacement is equal to 0, because A ends up back where it started. To find the acceleration of A while it is on the slope we have the following diagram.



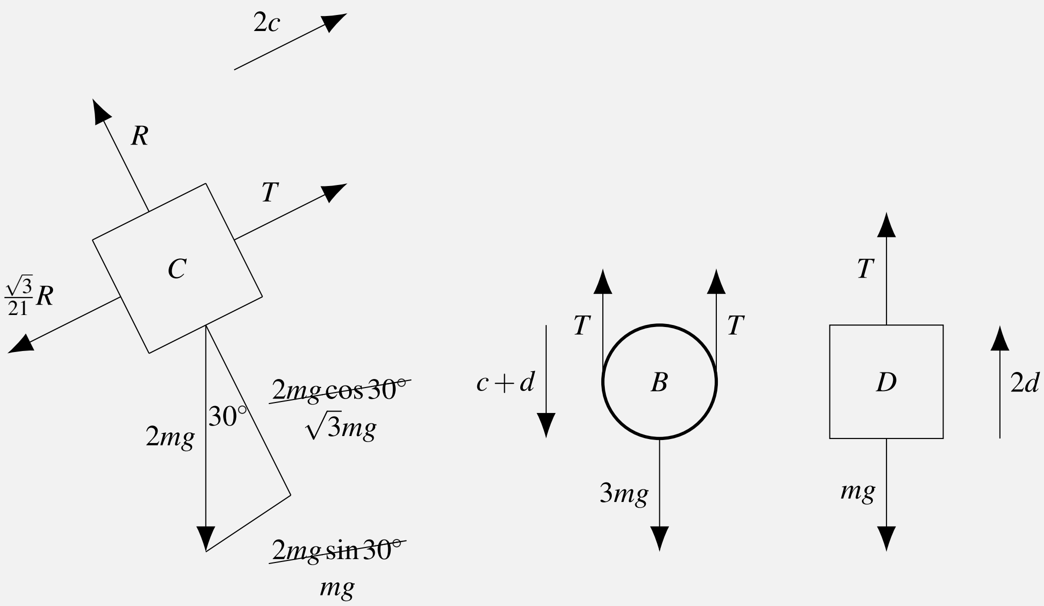
$$\begin{aligned} F &= ma \\ \Rightarrow -6mg &= 10ma \\ \Rightarrow -\frac{3}{5}g &= a. \end{aligned}$$

Then we have the UVAST array

$$\begin{aligned} u &= 0.98 \\ v &= \\ a &= -\frac{3}{5}g \\ s &= 0 \\ t &= \end{aligned}$$

$$s = ut + \frac{1}{2}at^2$$
$$\Rightarrow 0 = 0.98t - \frac{3}{10}gt^2$$
$$\Rightarrow t = \frac{1}{3} \text{ seconds.}$$

(b) (i)



Our equations are

$$R = \sqrt{3}mg$$
$$T - \frac{\sqrt{3}}{21}R - mg = 2m(2c)$$
$$3mg - 2T = 3m(c+d)$$
$$T - mg = m(2d).$$

Substituting our value for R into the second equation and using it to get mc in terms of T,

$$T - \frac{\sqrt{3}}{21}\sqrt{3}mg - mg = 4mc$$
$$\Rightarrow T - \frac{8mg}{7} = 4mc$$
$$\Rightarrow \frac{T}{4} - \frac{2mg}{7} = mc.$$

Using the fourth equation to get md in terms of T,

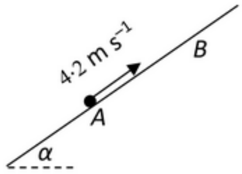
$$T - mg = 2md$$
$$\Rightarrow \frac{T}{2} - \frac{mg}{2} = md.$$

Substituting both of these expressions into the third equation,

$$
\begin{aligned}
3mg - 2T &= 3mc + 3md \\
\Rightarrow 3mg - 2T &= \frac{3T}{4} - \frac{6mg}{7} + \frac{3T}{2} - \frac{3mg}{2} \\
\Rightarrow 3mg + \frac{3mg}{2} + \frac{6mg}{7} &= \frac{3T}{2} + \frac{3T}{4} + 2T \\
\Rightarrow \frac{75mg}{14} &= \frac{17T}{4} \\
\Rightarrow \frac{150mg}{119} &= T.
\end{aligned}
$$

Question — 2019 Q1 (a).

1. **(a)** A particle P, of mass 3 kg, is projected along a rough inclined plane from the point A with speed $4{\cdot}2$ m s^{-1}. The particle comes to instantaneous rest at B.
The plane is inclined at an angle α to the horizontal where $\tan\alpha = \frac{9}{40}$.
The coefficient of friction between the particle and the plane is $\frac{3}{20}$.



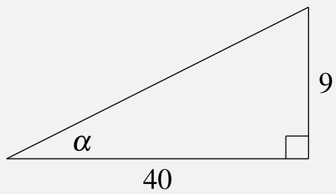
(i) Show that the deceleration of P is $\frac{15g}{41}$.

(ii) Find $|AB|$.

After reaching B the particle slides back down the plane.

(iii) Find the speed of P as it passes through A on its way back down the plane.

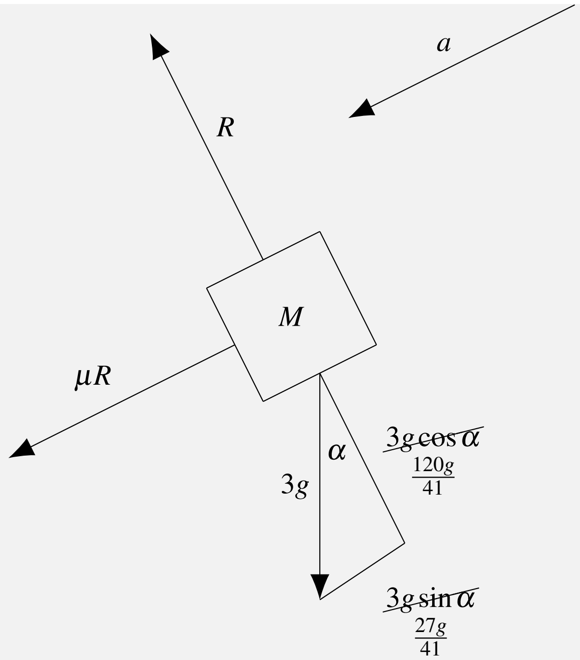
(i) If $\tan\alpha = \frac{9}{40}$, we can draw α inside the triangle



We can use Pythagoras' Theorem to calculate the hypotenuse to be 41, so that

$$\sin\alpha = \frac{9}{41}$$
$$\cos\alpha = \frac{40}{41}.$$

The forces acting on the particle are then as follows.



Note that although acceleration is down the slope, friction is also down the slope as **velocity** is up the slope. Then our two equations are

$$\begin{aligned} R &= \frac{120g}{41}, \\ \mu R + \frac{27g}{41} &= 3a \\ \Rightarrow \left(\frac{3}{20}\right)\left(\frac{120g}{41}\right) + \frac{27g}{41} &= 3a \\ \Rightarrow \frac{45g}{41} &= 3a \\ \Rightarrow \frac{15g}{41} &= a. \end{aligned}$$

(ii) We have, with no trigonometry required, the UVAST array

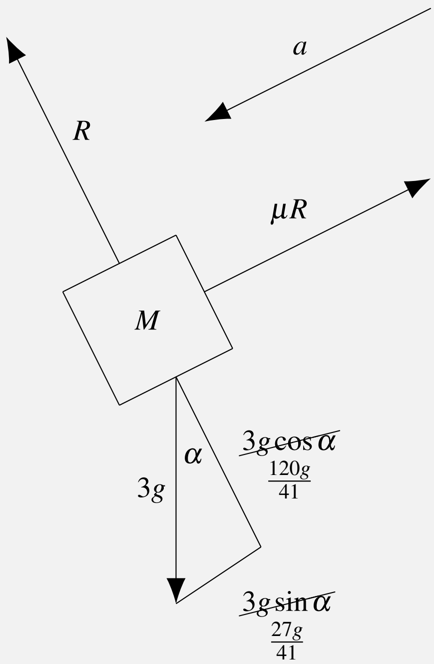
$$\begin{aligned} u &= 4.2 \\ v &= 0 \\ a &= -\frac{15g}{41} \\ s &= \\ t &= \end{aligned}$$

Then

$$\begin{aligned} v^2 &= u^2 + 2as \\ \Rightarrow 0 &= 17.64 - \frac{30g}{41}s \\ \Rightarrow s &= 2.46 \text{ m.} \end{aligned}$$

Therefore $|AB| = 2.46$ m.

(iii) On the way down the frictional force reverses direction and so the forces acting on the particle are as follows.



Therefore our equations are

$$R = \frac{120g}{41}$$
$$\frac{27g}{41} - \mu R = 3a$$
$$\Rightarrow \frac{27g}{41} - \left(\frac{3}{20}\right)\left(\frac{120g}{41}\right) = 3a$$
$$\Rightarrow \frac{9g}{41} = 3a$$
$$\Rightarrow \frac{3g}{41} = a.$$

Looking at the journey back from B to A we have the UVAST array

$$u = 0$$
$$v =$$
$$a = \frac{3g}{41}$$
$$s = 2.46$$
$$t =$$

Then

$$\begin{aligned} v^2 &= u^2 + 2as \\ &= 3.528 \\ \Rightarrow v &\approx 1.88 \text{ m/s.} \end{aligned}$$

Question — 2019 Q4 (a).

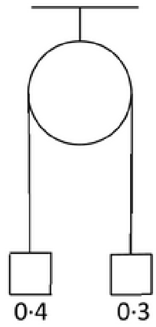
4. **(a)** Two particles of masses 0·4 kg and 0·3 kg are attached to the ends of a light inextensible string which passes over a light smooth fixed pulley. They are held at the same level, as shown in the diagram.

The system is released from rest.

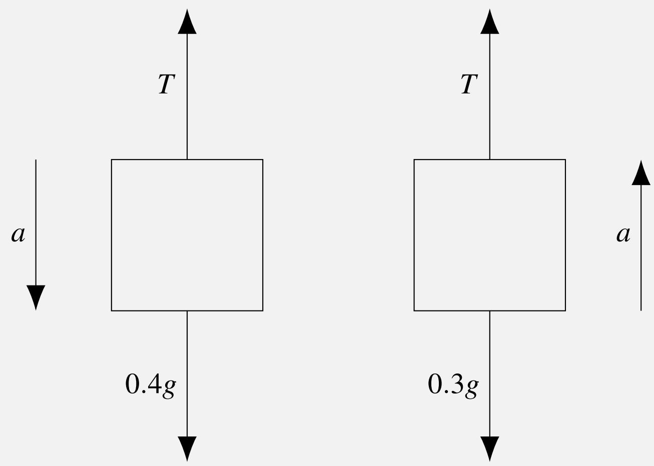
Find

(i) the tension in the string

(ii) the speed of the 0·4 kg mass when it has descended 0·7 m.



(i)



Our equations are then

$$0.4g - T = 0.4a$$
$$T - 0.3g = 0.3a$$

Adding these equations gives

$$0.1g = 0.7a$$
$$\Rightarrow \frac{g}{7} = a.$$

Using the second equation

$$T - 0.3g = 0.3\left(\frac{g}{7}\right)$$
$$\Rightarrow T = \frac{12}{35}g.$$

(ii) As the particles start at rest we have the following UVAST array for the 0.4 kg particle.

$$
\begin{aligned}
u &= 0 \\
v &= \\
a &= \frac{g}{7} \\
s &= 0.7 \\
t &=
\end{aligned}
$$

$$
\begin{aligned}
v^2 &= u^2 + 2as \\
&= 0 + \frac{g}{5} \\
\Rightarrow v &= \sqrt{\frac{g}{5}} \\
&= 1.4 \text{ m.}
\end{aligned}
$$

Question — 2018 Q1 (a).

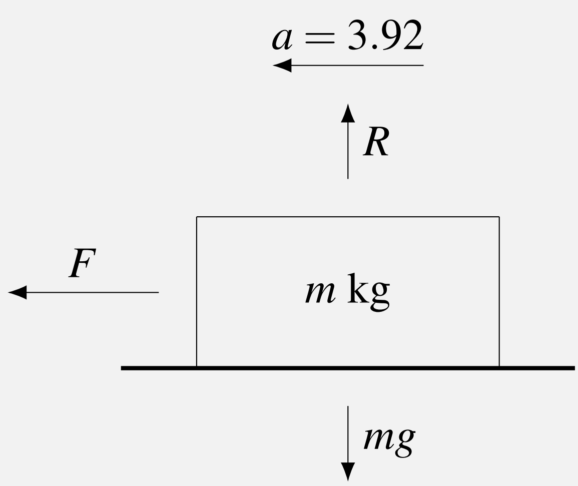
(a) A parcel rests on the horizontal floor of a van.
The van is travelling on a level road at 14 m s^{-1} .
It is brought to rest by a uniform application of the brakes.
The coefficient of friction between the parcel and the floor is $\frac{2}{5}$.
Show that the parcel is on the point of sliding forward on the floor of the van if the stopping distance is 25 m.

We first find the deceleration of the van.

$$\begin{aligned} u &= 14 \\ v &= 0 \\ a &= \\ s &= 25 \\ t &= \end{aligned}$$

$$\begin{aligned} v^2 &= u^2 + 2as \\ \Rightarrow 0 &= 14^2 + 50a \\ \Rightarrow -50a &= 196 \\ \Rightarrow a &= -3.92 \text{ m/s}^2. \end{aligned}$$

Friction is acting to the back of the van, pulling the parcel back. Assume that the particle is not moving along the floor. Then it is also decelerating at 3.92 m/s^2 with the van. As friction (F), reaction force (R) and gravity are the only forces acting on the parcel, we have the following diagram.



Then $R = mg$, and

$$\begin{aligned} F &= ma \\ \Rightarrow F &= 3.92m. \end{aligned}$$

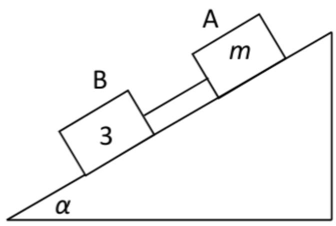
Also,

$$\begin{aligned} \mu R &= \frac{2}{5}mg \\ &= 3.92m, \end{aligned}$$

so if the particle is not moving on the floor friction (F) is equal to limiting friction (μR), proving the statement.

Question — 2018 Q4.

(a) A block A of mass m is connected by a light inextensible string to a second block B of mass 3 kg.
They slide down a rough inclined plane which makes an angle α with the horizontal where $\tan\alpha = \frac{3}{4}$.
The string remains taut in the subsequent motion.
The coefficient of friction between A and the plane is $\frac{3}{4}$.
The coefficient of friction between B and the plane is $\frac{1}{3}$.
The system is released from rest.

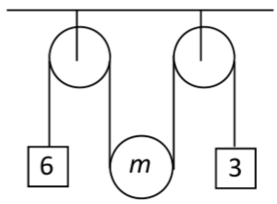


Find

(i) the acceleration of B, in terms of m

(ii) the value of m if the tension in the string is 3·92 N.

(b) A moveable pulley of mass m is suspended on a light inextensible string between two fixed pulleys as shown in the diagram. Masses of 6 kg and 3 kg are attached to the ends of the string.
The system is released from rest.

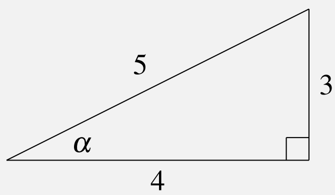


(i) Show, on separate diagrams, the forces acting on the moveable pulley **and** on each of the masses.

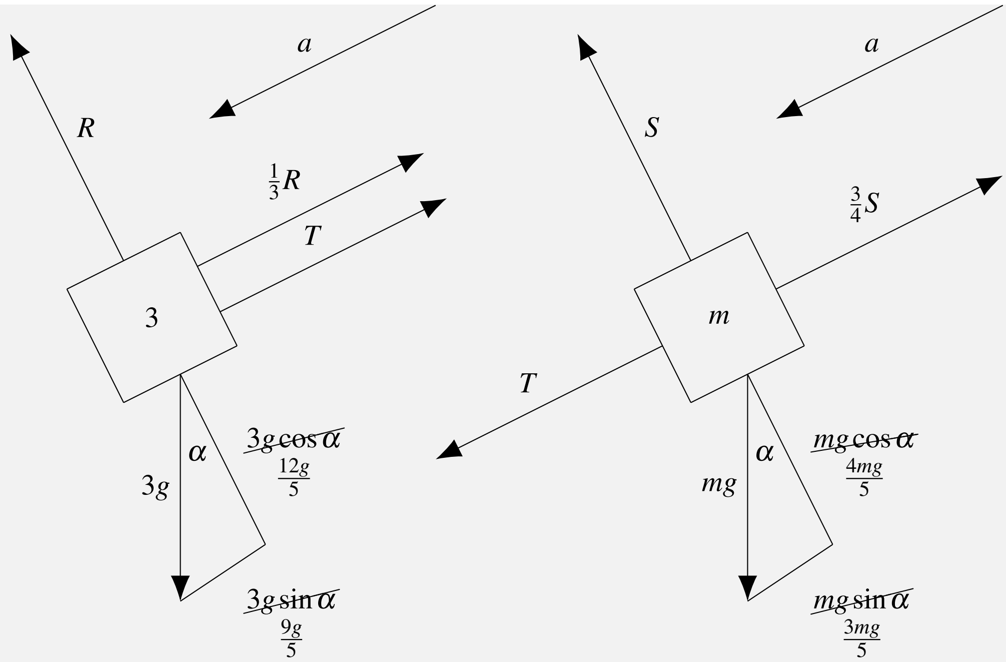
(ii) Find in terms of m the tension in the string.

(iii) For what value of m will the acceleration of the moveable pulley be zero?

(a) (i) By drawing a triangle and applying Pythagoras' Theorem, we can use $\tan\alpha = \frac{3}{4}$ to find that $\cos\alpha = \frac{4}{5}$, $\sin\alpha = \frac{3}{5}$.



Then our forces are



Our equations are

$$R = \frac{12g}{5}$$
$$\frac{9g}{5} - \frac{1}{3}R - T = 3a$$
$$S = \frac{4mg}{5}$$
$$T + \frac{3mg}{5} - \frac{3}{4}S = ma.$$

Substituting R and S into the second and fourth equation give us and then adding them we get

$$\frac{9g}{5} - \frac{4g}{5} - T = 3a$$
$$\Rightarrow g - T = 3a$$

and

$$T + \frac{3mg}{5} - \frac{3mg}{5} = ma$$
$$\Rightarrow T = ma.$$

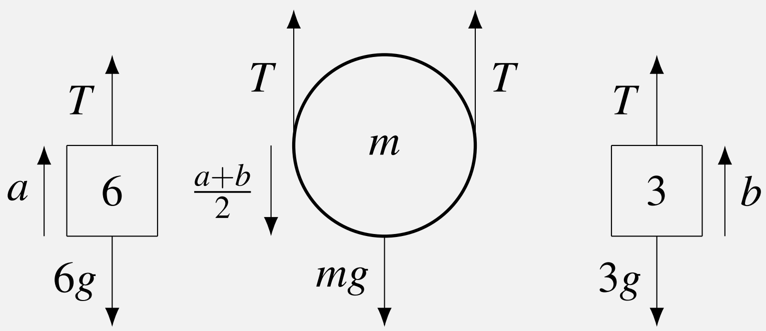
Substituting $T = ma$ into $g - T = 3a$ gives

$$g - ma = 3a$$
$$\Rightarrow g = (3+m)a$$
$$\Rightarrow \frac{g}{3+m} = a.$$

(ii) If $T = 3.92$ and $a = \frac{g}{3+m}$ then $T = ma$ becomes

$$\begin{aligned} 3.92 &= \frac{mg}{3+m} \\ \Rightarrow 11.76 + 3.92m &= 9.8m \\ \Rightarrow 11.76 &= 5.88m \\ \Rightarrow 2 \text{ kg} &= m. \end{aligned}$$

(b) (i) Note that the directions of the acceleration are note necessary for full marks, but they will be used later.



(ii) Our equations are

$$\begin{aligned} T - 6g &= 6a, \\ mg - 2T &= m\frac{a+b}{2}, \\ T - 3g &= 3b. \end{aligned}$$

Rewriting the first and third equations as

$$\begin{aligned} \frac{T}{6} - g &= a, \\ \frac{T}{3} - g &= b \end{aligned}$$

and plugging them into the second equation,

$$\begin{aligned} mg - 2T &= m\frac{a+b}{2} \\ \Rightarrow 2mg - 4T &= m(a+b) \\ &= m\left(\frac{T}{6} - g + \frac{T}{3} - g\right) \\ &= m\left(\frac{T}{2} - 2g\right) \\ &= \frac{mT}{2} - 2mg \\ \Rightarrow 4mg &= 4T + \frac{mT}{2} \\ \Rightarrow 8mg &= 8T + mT \\ &= (4+m)T \\ \Rightarrow \frac{8mg}{8+m} &= T. \end{aligned}$$

(iii) If the acceleration of the movable pulley is 0 then its upwards forces equal downwards forces and

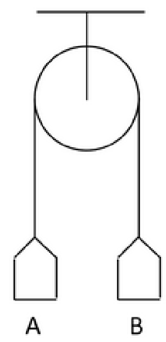
$$\begin{aligned} 2T &= mg \\ \Rightarrow \frac{16mg}{8+m} &= mg \\ \Rightarrow \frac{16}{8+m} &= 1 \\ \Rightarrow 16 &= 8+m \\ \Rightarrow 8 &= m. \end{aligned}$$

Question — 2017 Q4 (a).

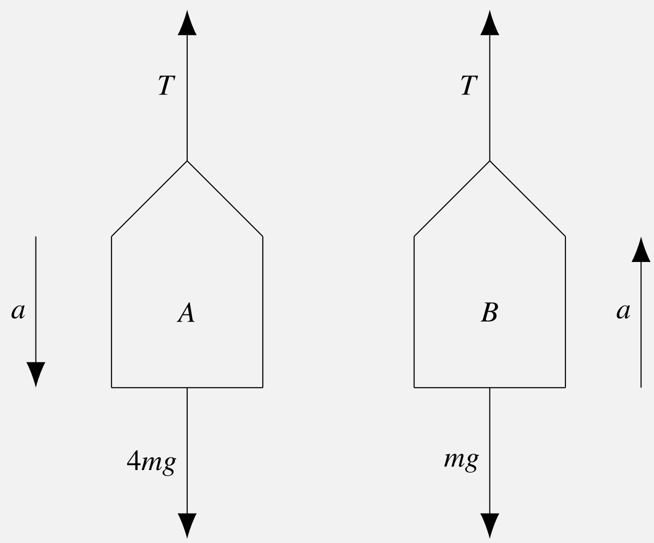
4. **(a)** Two scale pans A and B, each of mass m kg, are attached to the ends of a light inextensible string which passes over a light smooth fixed pulley. They are held at the same level, as shown in the diagram.
A mass of $3m$ kg is now placed on A.

The system is released from rest.

Find
(i) the tension in the string in terms of m
(ii) how far B has risen when it reaches a speed of 0·4 m s^{-1}
(iii) the reaction on the $3m$ kg mass in terms of m.



(i) For parts (i) and (ii) we can treat the scale pan A and the $3m$ mass as a single object of mass $4m$. Our diagram is then as follows.



Our equations are then

$$4mg - T = 4ma$$
$$T - mg = ma$$

Substituting ma from the second equation into the first gives

$$\begin{aligned} 4mg - T &= 4(T - mg) \\ &= 4T - 4mg \\ \Rightarrow 8mg &= 5T \\ \Rightarrow \frac{8mg}{5} &= T. \end{aligned}$$

(ii) If $T = \frac{8mg}{5}$ then

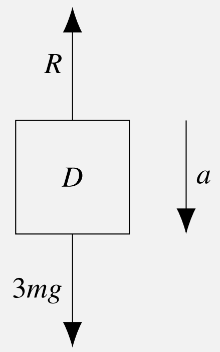
$$\begin{aligned} T - mg &= ma \\ \Rightarrow \frac{8mg}{5} - mg &= ma \\ \Rightarrow \frac{3mg}{5} &= ma \\ \Rightarrow \frac{3g}{5} &= a. \end{aligned}$$

The journey for B has UVAST array

$$\begin{aligned} u &= 0 \\ v &= 0.4 \\ a &= \frac{3g}{5} \\ s &= \\ t &= \end{aligned}$$

$$\begin{aligned} v^2 &= u^2 + 2as \\ \Rightarrow 0.4^2 &= 0^2 + \frac{6g}{5}s \\ \Rightarrow 0.014 \text{ m} &= s. \end{aligned}$$

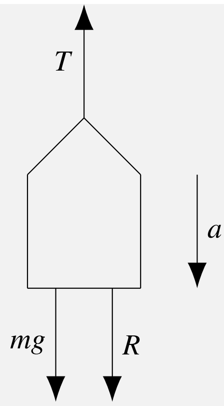
(iii) Consider the forces acting on the $3m$ mass.



As we already know a,

$$\begin{aligned} 3mg - R &= 3m\left(\frac{3g}{5}\right) \\ \Rightarrow 3mg - \frac{9mg}{5} &= R \\ \frac{6mg}{5} &= R. \end{aligned}$$

Alternatively we could have looked at the forces acting on the scale pan A.

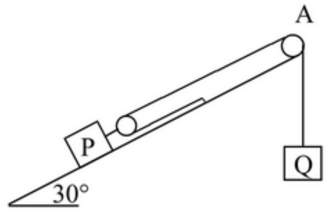


As we already know a and T,

$$\Rightarrow R + mg - T = ma$$
$$\Rightarrow R + mg - \frac{8mg}{5} = \frac{3mg}{5}$$
$$\Rightarrow R = \frac{6mg}{5}.$$

Question — 2016 Q4.

4. **(a)** The block P has a light pulley fixed to it. The two blocks P and Q, of mass 40 kg and 30 kg respectively, are connected by a taut light inextensible string passing over a light smooth fixed pulley, A, as shown in the diagram.

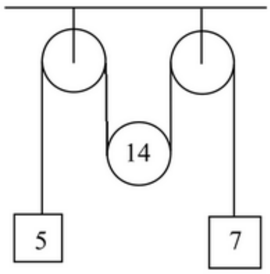


P is on a rough plane which is inclined at 30° to the horizontal. The coefficient of friction between P and the inclined plane is $\frac{1}{4}$.

Q is hanging freely. The system is released from rest.

Find
(i) the acceleration of P and the acceleration of Q
(ii) the speed of P when it has moved 30 cm.

(b) A light inextensible string passes over a small smooth fixed pulley, under a small smooth moveable pulley, of mass 14 kg, and then over a second small smooth fixed pulley. A 5 kg mass is attached to one end of the string and a 7 kg mass is attached to the other end.



The system is released from rest.
(i) Find the tension in the string.
(ii) If instead of the system starting from rest, the moveable pulley is given an initial upward velocity of $0{\cdot}8$ m s^{-1}, find the time taken until the moveable pulley reverses direction.

(a) (i) If particle P has acceleration a then particle Q has acceleration $2a$. Our diagram of forces are then

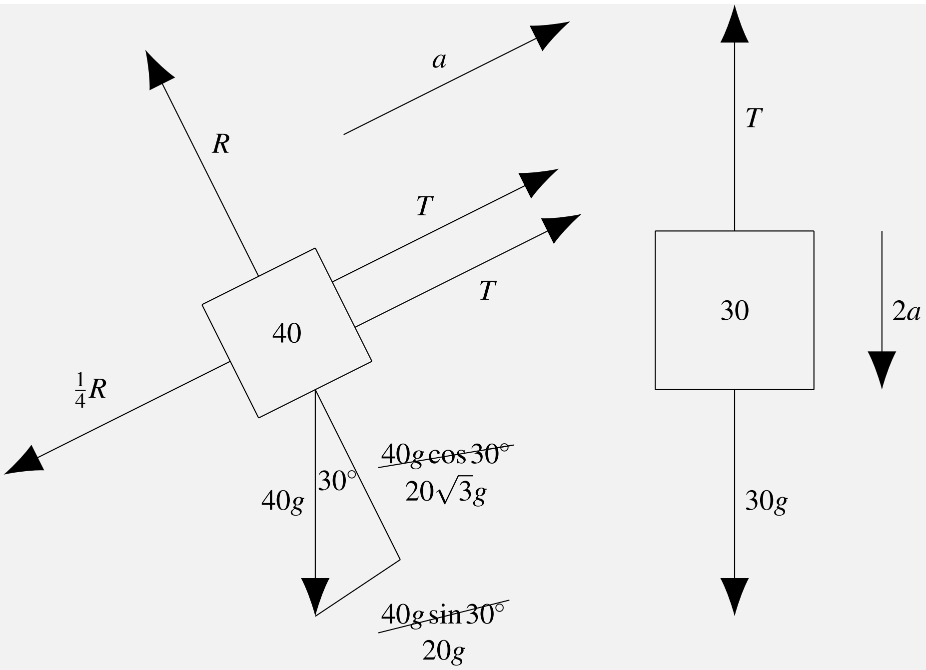


Figure 1

Our equations are

$$R = 20\sqrt{3}g$$

$$T - \frac{1}{4}R - 20g = 40a,$$

$$30g - T = 30(2a).$$

Rewriting the third equation as $30g - 60a = T$, and using the third and first equation to replace R and T in the second equation gives us

$$2(30g - 60a) - \frac{1}{4}(20\sqrt{3}g) - 20g = 40a$$

$$\Rightarrow 60g - 120a - 5\sqrt{3}g - 20g = 40a$$

$$\Rightarrow 40g - 5\sqrt{3}g = 160a$$

$$\Rightarrow \frac{40 - 5\sqrt{3}}{160}g = a$$

$$\Rightarrow 1.92 \text{ m/s}^2 = a$$

(ii) The UVAST array for this journey is

$$u = 0$$

$$v =$$

$$a = 1.92$$

$$s = 0.3$$

$$t =$$

$$v^2 = u^2 + 2as$$
$$= 1.152$$
$$\Rightarrow v = 1.07 \text{ m/s.}$$

(b) (i) Our forces and accelerations are as follows.

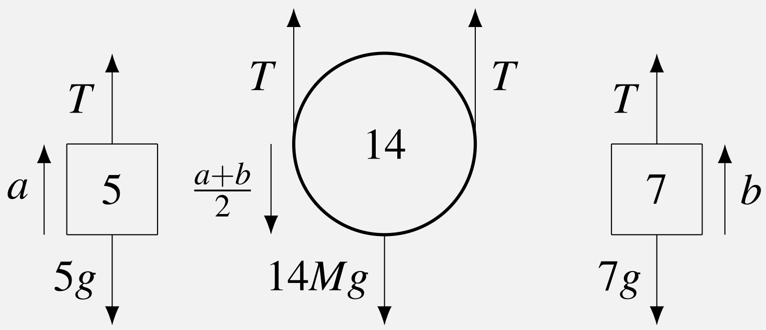


Figure 2

This gives us the equations

$$T - 5g = 5a$$
$$14g - 2T = 14\left(\frac{a+b}{2}\right)$$
$$T - 7g = 7b.$$

Rewriting the first equation with a as the subject,

$$\frac{T}{5} - g = a.$$

Rewriting the third equation with b as the subject,

$$\frac{T}{7} - g = b.$$

Simplifying the second equation first and then replacing a and b,

$$14Mg - 2T = 14\left(\frac{a+b}{2}\right)$$
$$= 7a + 7b$$
$$\Rightarrow 14g - 2T = 7\left(\frac{T}{5} - g\right) + 7\left(\frac{T}{7} - g\right)$$
$$= \frac{7T}{5} - 7g + T - 7g$$
$$\Rightarrow 28g = \frac{22T}{5}$$
$$\Rightarrow \frac{70g}{11} = T.$$

(ii) First, if $T = \frac{70g}{11}$,

$$\begin{aligned} a &= \frac{T}{5} - g \\ &= \frac{3g}{11}, \\ b &= \frac{T}{7} - g \\ &= -\frac{g}{11}. \end{aligned}$$

The change in initial velocity doesn't change the fact that the movable pulley initially has acceleration

$$\frac{a+b}{2} = \frac{g}{11}$$

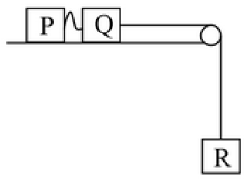
downwards. This gives the UVAST array

$$\begin{aligned} u &= 0.8 \\ v &= 0 \\ a &= -\frac{g}{11} \\ s &= \\ t &= \end{aligned}$$

$$\begin{aligned} v &= u + at \\ \Rightarrow 0 &= 0.8 - \frac{g}{11}t \\ \Rightarrow \frac{g}{11}t &= 0.8 \\ \Rightarrow t &= 0.9 \text{ seconds.} \end{aligned}$$

Question — 2015 Q4 (a).

(a) Two particles P and Q, of mass 4 kg and 7 kg respectively, are lying 0.5 m apart on a smooth horizontal table. They are connected by a string 3.5 m long. Q is 6 m from the edge of the table and is connected to a particle R, which is of mass 3 kg and is hanging freely, by a taut light inextensible string passing over a light smooth pulley.



The system is released from rest.

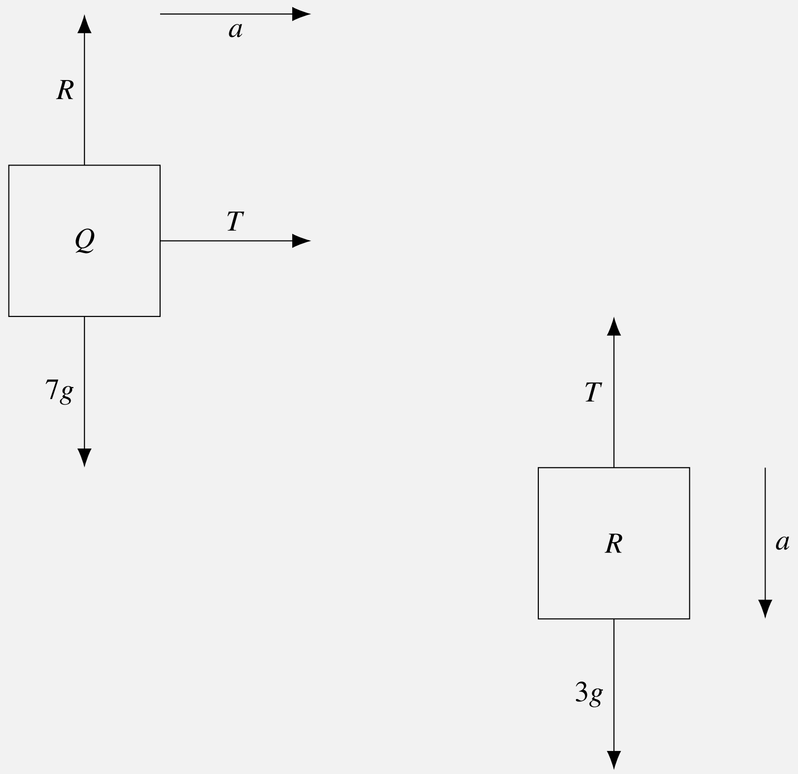
Find

(i) the initial acceleration of Q and R

(ii) the speed of Q when it has moved 3 m

(iii) the speed with which P begins to move.

(i) As the PQ string is slack it originally provides no tensile force to Q.



Our equations are

$$\begin{aligned} R &= 7g \\ T &= 7a \\ 3g - T &= 3a \end{aligned}$$

Ignoring the first equation and adding the second and third gives us

$$\begin{aligned} 3mg &= 10a \\ \Rightarrow \frac{3g}{10} &= a. \end{aligned}$$

(ii)

$$\begin{aligned} u &= 0 \\ v &= \\ a &= \frac{3g}{10} \\ s &= 3 \\ t &= \end{aligned}$$

$$\begin{aligned} v^2 &= u^2 + 2as \\ \Rightarrow v &= \sqrt{\frac{9g}{5}} \\ &= 4.2 \text{ m/s}. \end{aligned}$$

(iii) The PQ string goes taut when Q has travelled 3 m. By part (ii) its speed is 4.2 m/s at this time. The moment the string goes taut can be treated as a collision between P and the joint mass that is Q and R; the principle of conservation of momentum applies. Letting v be their joint velocity after the "collision"

$$\begin{aligned} u_1m_1 + u_2m_2 &= v_1m_1 + v_2m_2 \\ \Rightarrow 0(4) + 4.2(10) &= 4v + 10v \\ \Rightarrow 42 &= 14v \\ \Rightarrow 3 \text{ m/s} &= v. \end{aligned}$$