

Note These exam questions are given in reverse chronological order as they appear in exam papers; 2023 paper, Sample paper, 2022 (deferred), 2022, and so on back to 2015. As the collisions portion of the syllabus is unchanged no questions from old syllabus exam papers have been excluded.

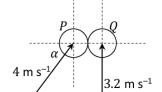
Question — 2023 **Q3**.

(b) Two smooth spheres, P and Q, have equal radius and are of mass m and 2m respectively. P and Q collide obliquely. The line joining their centres at the point of impact lies along the $\vec{\iota}$ axis.



Before the collision, sphere P moves with a velocity of 4 m s⁻¹ at an angle α with the $\vec{\iota}$ axis, where $\sin \alpha = \frac{4}{\varsigma}$.

Before the collision, sphere Q moves with a velocity of 3.2 m $\rm s^{-1}$ perpendicular to the \vec{t} axis.



The coefficient of restitution between the spheres is e , where $0 \le e \le 1$.

Calculate, in terms of e, the velocity of each sphere immediately after they collide

Question — Sample Q4.

Question 4

(a) A ball is projected from a point on horizontal ground, with initial speed u and at an angle α to the horizontal. The ball reaches a maximum height of H_0 above the horizontal.

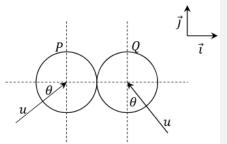
Upon landing, the ball bounces with a maximum height of H_1 .



The coefficient of restitution between the ball and the ground is e.

- (i) Calculate $\frac{H_0}{H_1}$.
- (ii) The ball continues bouncing. Find an expression (in terms of e and H_0) for H_5 , the maximum height of the ball after it lands on the ground for the fifth time.
- (b) Two identical smooth spheres, P and Q, each moving with speed u, collide obliquely. The line joining their centres at the point of impact is along the \vec{i} axis.

Before the collision, the velocity of sphere P makes an angle θ with the $\vec{\iota}$ axis and the velocity of sphere Q makes an angle θ with the $\vec{\jmath}$ axis, as shown in the diagram.



The coefficient of restitution between the spheres is e, where $0 \le e \le 1$.

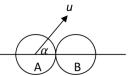
After the collision sphere Q moves off parallel to the \vec{j} axis.

- (i) Show that $e = \frac{\tan \theta 1}{\tan \theta + 1}$.
- (ii) If 25% of the spheres' total kinetic energy is lost during the collision, calculate θ and e.

Question — 2022 (Deferred) Q5.

- **5. (a)** A smooth sphere, P, of mass 3*m* collides directly with another smooth sphere, Q, of mass 5*m*. P and Q are moving in opposite directions before impact with speeds 4*u* and 2*u* respectively. The coefficient of restitution for the collision is *e*.
 - (i) Find the speed of P and the speed of Q after impact in terms of u and e.
 - (ii) If P and Q are moving in the same direction after impact, show that $0 \le e < \frac{1}{15}$.
 - **(b)** A smooth sphere, A, of mass *m* collides obliquely with another smooth sphere, B, of mass *m*.

Before impact, A is moving with speed u at an angle α to the line of centres of the spheres, where $0^{\circ} < \alpha < 45^{\circ}$.



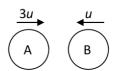
B is at rest before the impact.

The coefficient of restitution for the collision is e.

- (i) Find the speed of A and the speed of B after impact in terms of u, e and α .
- (ii) Given that A is deflected through angle α because of the collision, show that $\tan^2 \alpha = e$.

Question — 2022 Q5.

(a) A smooth sphere A of mass 2m, moving with speed 3u on a smooth horizontal table collides directly with a smooth sphere B of mass m, moving in the opposite direction with speed u.



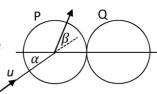
The coefficient of restitution between A and B is e.

Find, in terms of u and e,

- (i) the speed of each sphere after the collision
- (ii) the magnitude of the impulse imparted to B due to the collision.

The loss of the kinetic energy due to the collision is $kmu^2(1-e^2)$.

- (iii) Find the value of k.
- (b) A smooth sphere P has mass m and speed u. It collides obliquely with a smooth sphere Q, of mass m, which is at rest. Before the collision, the direction of P makes an angle α with the line of centres, as shown in the diagram.



The coefficient of restitution between the spheres is $\frac{1}{3}$.

During the impact the direction of motion of P is turned through an angle β .

Show that
$$\tan \beta = \frac{2 \tan \alpha}{1 + 3 \tan^2 \alpha}$$

Question — 2021 Q5.

(a) A smooth sphere A of mass 4m, moving with speed u on a smooth horizontal table collides directly with a smooth sphere B of mass m, moving in the opposite direction with speed u.

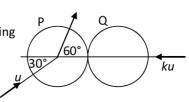
A B

The coefficient of restitution between A and B is e.

(i) Find the speed, in terms of *u* and *e*, of each sphere after the collision.

The magnitude of the impulse on B due to the collision is T.

- (ii) Show that $\frac{8mu}{5} \le T \le \frac{16mu}{5}$.
- (b) A smooth sphere P has mass 2m and speed u. It collides obliquely with a smooth sphere Q of mass m which is moving with speed ku, as shown in the diagram.
 Before the collision, the direction of P makes an angle of 30° to the line of centres. After the collision, the direction of P makes an angle of 60° to the line of



The coefficient of restitution between the spheres is e.

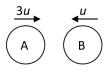
- (i) Show that $k = \frac{\sqrt{3}(1-e)}{2(1+e)}$.
- (ii) Find the speed of Q immediately after the collision.

Question — 2020 Q5.

centres.

(a) A smooth sphere A of mass m, moving with speed 3u on a smooth horizontal table collides directly with a smooth sphere B of mass 2m, moving in the opposite direction with speed u.

The directions of motion of A and B are reversed by the collision.

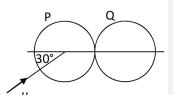


The coefficient of restitution between A and B is e.

(i) Find the speed, in terms of u and e, of each sphere after the collision. Subsequently B hits a wall at right angles to the line of motion of A and B. The coefficient of restitution between B and the wall is $\frac{1}{2}$.

After B rebounds from the wall there is a further collision between A and B.

- (ii) Show that $\frac{1}{8} < e < \frac{1}{4}$.
- (b) A smooth sphere P has mass m_1 and speed u. It collides obliquely with a smooth sphere Q, of mass m_2 , which is at rest.



Before the collision the direction of P makes an angle of 30° to the line of centres, as shown in the diagram.

The coefficient of restitution between the spheres is e.

Prove that P will turn through a right-angle if $4m_1 = (3e - 1)m_2$.

Question — 2019 Q5.

- (a) A small smooth sphere A, of mass 3m moving with speed u, collides directly with a small smooth sphere B, of mass m moving with speed u in the opposite direction. The coefficient of restitution between the spheres is $\frac{1}{2}$.
 - (i) Find, in terms of u, the speed of each sphere after the collision.

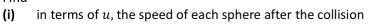
After the collision B hits a smooth vertical wall which is perpendicular to the direction of motion of B. The coefficient of restitution between B and the wall is $\frac{2}{5}$.

The first collision between the spheres occurred at a distance 2 metres from the wall. The spheres collide again 4 seconds after the first collision between them.

- (ii) Find the value of u.
- (b) A smooth sphere P, of mass 2m, collides with a smooth sphere Q, of mass m. The velocity of P is $3u \ \vec{i} + 4u \ \vec{j}$ and the velocity of Q is $-4u \ \vec{i} + 3u \ \vec{j}$, where \vec{i} is along the line of centres at impact.

The coefficient of restitution between the spheres is $\frac{5}{7}$.

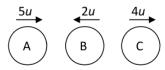
Find





Question — 2018 Q5.

(a) Three identical small smooth spheres A, B and C, each of mass m, lie in a straight line on a smooth horizontal surface with B between A and C. Spheres A and B are projected towards each other with speeds 5u and 2u respectively, and at the same



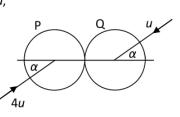
time C is projected along the line from B away from B with speed 4u.

The coefficient of restitution between each pair of spheres is *e*.

After the collision between A and B there is a collision between B and C.

(i) Find, in terms of *e* and *u*, the speed of each sphere after the first collision.

- (ii) Show $e > \frac{5}{7}$.
- (iii) If $e = \frac{6}{7}$ show that B will not collide with A again.
- (b) A small smooth sphere P, of mass 2m, moving with speed 4u, collides obliquely with an equal smooth sphere Q, of mass 3m, moving with speed u. Before the collision the spheres are moving in opposite directions, each making an angle α to the line of centres, as shown in the diagram.



The coefficient of restitution between the spheres is $\frac{1}{5}$.

(i) Find, in terms of u and α , the speed of each sphere after the collision.

After the collision the speed of P is twice the speed of Q.

(ii) Find the value of α .

Question — 2017 Q5.

(a) A small smooth sphere A, of mass 1.5 kg, moving with speed 6 m s⁻¹, collides directly with a small smooth sphere B, of mass m kg, which is at rest. After the collision the spheres move in opposite directions with speeds v and 2v, respectively.

80% of the kinetic energy lost by A as a result of the collision is transferred to B. The coefficient of restitution between the spheres is e.

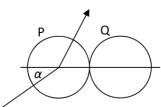
Find (i) the value of v

(ii) the value of e.

(b) A small smooth sphere P, of mass 3*m*, collides obliquely with a small smooth sphere Q, of mass 7*m*, which is at rest.

Before the collision the velocity of P makes an angle α with the line joining the centres of the spheres. After the collision the speed of Q is ν .

The coefficient of restitution between the spheres is $\frac{2}{7}$.



- (i) Find, in terms of v and α , the **speed** of P before the collision.
- (ii) If $\alpha=30^\circ$ find the angle through which the direction of motion of P is deflected as a result of the collision.

Question — 2016 Q5.

(a) Two small smooth spheres A, of mass 2 kg, and B, of mass 3 kg, are suspended by light strings from a ceiling as show in the diagram. The distance from the ceiling to the centre of each sphere is 2 m.

(A) (B) (C)

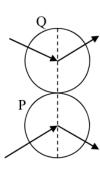
Sphere A is drawn back 60° and released from rest. A collides with B and rebounds. B swings through an angle θ .

The coefficient of restitution between the spheres is $\frac{3}{4}$.

- (i) Show that A strikes B with a speed of $\sqrt{2g}$ m s⁻¹.
- (ii) Find the speed of each sphere after the collision.
- (iii) Find the value of θ .
- (b) Two identical smooth spheres P and Q collide.

The velocity of P **after** impact is $3\vec{i} - \vec{j}$ and the velocity of Q **after** impact is $2\vec{i} + \vec{j}$, where \vec{j} is along the line of the centres of the spheres at impact.

The coefficient of restitution between the spheres is $\frac{1}{2}$.

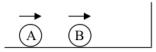


Find

- (i) the velocities, in terms of \vec{i} and \vec{j} , of the two spheres before impact
- (ii) to the nearest degree, the angle through which the direction of motion of P is deflected by the collision.

Question — 2015 Q5.

(a) A small smooth sphere A, of mass 2m, moving with speed $9u \text{ m s}^{-1}$, collides directly with a small smooth sphere B, of mass 5m, which is moving in the same direction with speed $2u \text{ m s}^{-1}$.



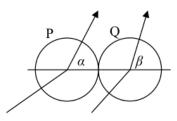
Sphere B then collides with a vertical wall, rebounds and collides again with sphere A. The wall is perpendicular to the direction of motion of the spheres.

The first collision takes place 35 cm from the wall.

The coefficient of restitution between the spheres is $\frac{4}{5}$.

The coefficient of restitution between sphere B and the wall is $\frac{5}{14}$.

- (i) Show that, as a result of the first collision, A comes to rest.
- (ii) Find the time between the two collisions between A and B in terms of u.
- **(b)** Two identical smooth spheres, P and Q, collide.



The coefficient of restitution is 1.

The velocity of P before impact is $a\vec{i} + b\vec{j}$ and the velocity of Q before impact is $c\vec{i} + d\vec{j}$, where \vec{i} is along the line of the centres of the spheres at impact.

After impact the direction of motion of P makes an angle α with their line of centres and the direction of motion of Q makes an angle β with their line of centres.

Show that $\tan \alpha \tan \beta = \frac{bd}{ac}$