Circular Motion Exam Questions

Note These exam questions are given in reverse chronological order as they appear in exam papers; 2023 paper, Sample paper, 2022 (deferred), 2022, and so on back to 2015. For the sake of including questions on horizontal circular motion due to friction exam questions from 2006 and 2000 are also included. Only questions from the old syllabus relevant to the new syllabus are included.

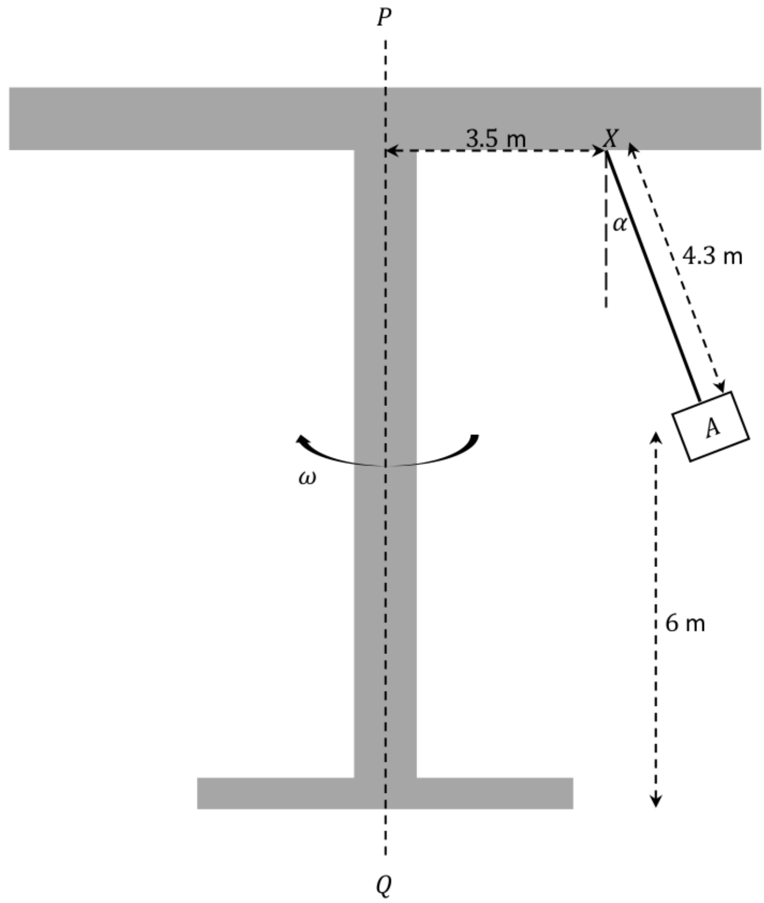
Question — 2023 Q3.

Question 3

The photograph on the right is of a chain swing ride in an amusement park. The disk at the top of the ride is rotating in a horizontal plane. People sit in seats which are attached freely by inextensible chains of length 4.3 m to fixed points on the disk.

The chain attaching seat A hangs from point X on the ride and makes an angle α with the vertical. X is 3.5 m from the axis of rotation, which is the vertical line PQ, as shown in the diagram below. The chain is free to swing in or out relative to PQ.

The ride rotates about PQ with constant angular velocity ω. Seat A moves in a horizontal circular path which is 6 m above the ground.



(i) Draw a diagram to show the external forces acting on seat A.

(ii) Show that $\omega = \sqrt{\frac{g \tan \alpha}{3.5+4.3 \sin \alpha}}$.

(iii) Use dimensional analysis to show that the units for the expression $\sqrt{\frac{g \tan \alpha}{3.5+4.3 \sin \alpha}}$ are equivalent to the units for ω.

It is found by measurement that $\alpha = 25°$.

(iv) Calculate how many complete revolutions the ride makes in one minute.

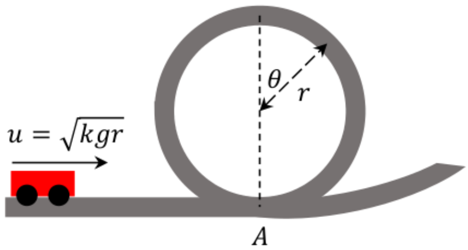
The person sitting in seat A throws a small orange into the air. The person imparts an upward vertical velocity component of $4\ \text{m s}^{-1}$ to the orange.

(v) Calculate the time from when the orange is thrown until it hits the ground.

Question — 2023 Q10 (b).

(b) A toy car track consists of a series of components that connect to make a closed circuit. Part of the track makes a vertical circular loop.

To model the motion of a car on this track, its velocity at the base of the loop (point A) is expressed as $u = \sqrt{kgr}$, where r is the radius of the loop, g is the acceleration due to gravity, and k is a constant.



The model ignores the effects of friction.

(i) Draw a diagram to show the forces acting on the car at the instant when the radius to the car makes an angle θ with the upward vertical.

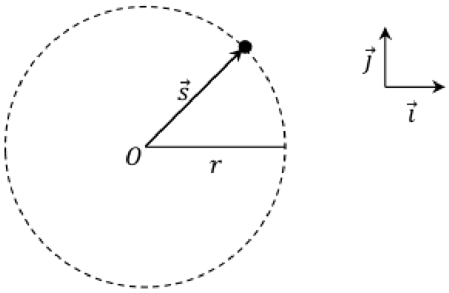
(ii) If the car loses contact with the track at the instant when the radius to the car makes an angle θ with the upward vertical, show that $\cos\theta = \frac{k-2}{3}$.

(iii) Calculate the minimum value of k such that the car successfully completes the loop without losing contact with the track.

Question — Sample Q6.

Question 6

A learner driver is practising driving around a roundabout.



The motion of the car may be modelled as horizontal circular motion around centre O, with radius r and constant angular speed ω, as in the diagram above.

(i) Write an expression for $\vec{s}$, the displacement of the car relative to O at any time t, in terms of r, ω and t. Your expression should use the unit vectors $\vec{\imath}$ and $\vec{\jmath}$.

Note that $t = 0$ when $\vec{s}$ is along the $\vec{\imath}$ axis.

(ii) Derive an expression for $\vec{v}$, the velocity of the car at any time t.

(iii) Use a dot product calculation to show that the car's velocity and displacement are always perpendicular to each other.

(iv) Show that the acceleration of the car is always directed towards O.

(v) Derive an expression for the maximum velocity the car could have as it travels around the roundabout, without slipping. Your expression should be written in terms of r, g and μ, the coefficient of friction between the car and the road.

(vi) Use dimensional analysis to show that the units for the expression you derived in part **(v)** are equivalent to the units for velocity.

(vii) Do you think the assumptions made in developing this model were appropriate? Explain your answer.

Question — 2022 (Deferred) Q6 (b).

(b) A particle of mass m is suspended vertically from a fixed point O by a light inelastic string of length d metres.

The particle is projected horizontally with speed u, where $u^2 = 4gd$.

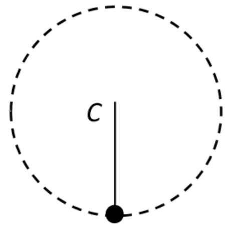
Show the string goes slack when it makes an angle $\cos^{-1}\frac{2}{3}$ with the upward vertical through O.

Question — 2022 Q6 (b).

(b) A particle is attached to one end of a light inextensible string of length 0.5 m. The other end of the string is attached to a fixed point C. The particle moves in a vertical circle.

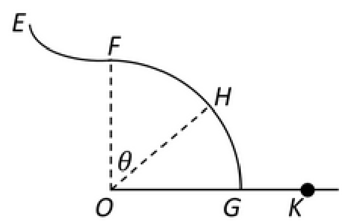
The greatest and least tensions in the string are $3T$ and T, respectively.

Find the speed of the particle at the lowest point.



Question — 2021 Q6 (b).

(b) A smooth slide EFG is in the shape of two arcs, EF and FG, each of radius r. The centre O of arc FG is vertically below F as shown in the diagram.
Point E is at a height $\frac{r}{5}$ above point F.
A child starts from rest at E, moves along the slide past the point F and loses contact with the slide at point H. OH makes an angle θ with the vertical.



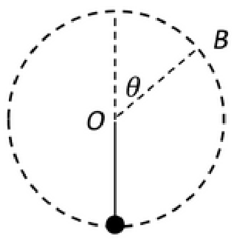
(i) Find the value of θ.

The child lands in a sandpit at point K.

(ii) Find, in terms of r, the speed of the child at K.

Question — 2020 Q6 (b).

(b) A particle P is attached to one end of a light inextensible string of length d.
The other end of the string is attached to a fixed point O.
The particle is hanging freely at rest, with the string vertical, when it is projected horizontally with speed $\sqrt{3gd}$.
The particle moves in a vertical circle.
The string becomes slack when P is at the point B.
OB makes an angle θ with the upward vertical.

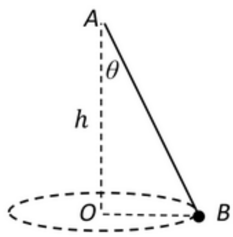


(i) Show that $\cos\theta = \frac{1}{3}$.

(ii) In terms of d, find the greatest height of P above B in the subsequent motion.

Question — 2019 Q6 (a).

6. (a) One end A of a light elastic string is attached to a fixed point. The other end, B, of the string is attached to a particle of mass m. The particle moves on a smooth horizontal table in a circle with centre O, where O is vertically below A and $|AO| = h$. The string makes an angle θ with the downward vertical and B moves with constant angular speed ω about OA.



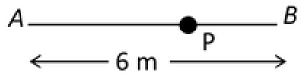
(i) Show that $\omega^2 \leq \frac{g}{h}$.

The elastic string has natural length h and elastic constant $\frac{2mg}{h}$.

(ii) Given that $\omega^2 = \frac{2g}{5h}$, find the value of θ.

Question — 2018 Q6 (ignoring (a)(ii)).

(a) Two points *A* and *B* are 6 m apart on a smooth horizontal surface. A particle P of mass 0·5 kg is attached to one end of a light elastic string, of natural length 2·5 m and elastic constant 8 N m^{-1}. The other end of the string is attached to *A*.



A second light elastic string, of natural length 1·5 m and elastic constant 12 N m^{-1} has one end attached to P and the other end attached to *B*, as shown in the diagram.
Initially P rests in equilibrium at the point *O*, where *AOB* is a straight line.

(i) Find the length of *AO*.

(b) A particle P is attached to one end of a light inextensible string of length d. The other end of the string is attached to a fixed point. The particle is hanging freely at rest, with the string vertical, when it is projected horizontally with speed u.
The particle moves in a complete vertical circle.

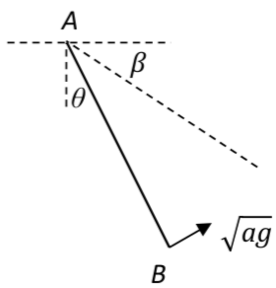
(i) Show that $u \geq \sqrt{5gd}$.

As P moves in the circle the least tension in the string is T_1 and the greatest tension is kT_1.

(ii) Given that $u = \sqrt{6gd}$, find the value of k.

Question — 2017 Q6 (b).

(b) One end *A* of a light inextensible string of length $3a$ is attached to a fixed point. A particle of mass *m* is attached to the other end *B* of the string. The string makes an angle θ with the vertical.



The particle is held in equilibrium with the string taut and $\cos\theta = \frac{2}{3}$. The particle is then projected with speed $\sqrt{ag}$, in the direction perpendicular to *AB*, as show in the diagram.
In the subsequent motion the string remains taut.

When *AB* makes an angle β below the horizontal, the speed of the particle is v and the tension in the string is *T*.

(i) Show that $v^2 = 3ag(2\sin\beta - 1)$.

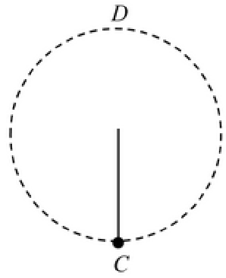
(ii) Find the minimum value and the maximum value of *T*.

Question — 2016 Q6 (a).

(a) A small particle hanging on the end of a light inextensible string 2 m long is projected horizontally from the point C.

(i) Calculate the least speed of projection needed to ensure that the particle reaches the point D which is vertically above C.

(ii) If the speed of projection is 7 m s^{-1} find the angle that the string makes with the vertical when it goes slack.



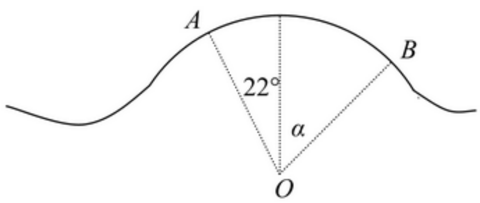
Question — 2015 Q6 (b).

(b) A skier of mass m kg is skiing on a hillside when he reaches a small hump in the form of an arc AB of a circle centre O and radius 7 m, as shown in the diagram.

O, A and B lie in a vertical plane and OA and OB make angles of 22° and α with the vertical respectively.

The skier's speed at A is 8 m s^{-1}.

The skier looses contact with the ground at point B. Find the value of α.

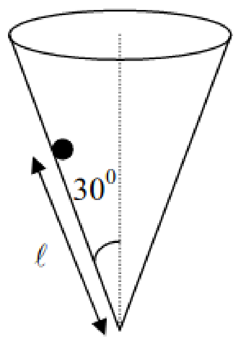


Question — 2006 Q6 (b).

A hollow cone with its vertex downwards and its axis vertical, revolves about its axis with a constant angular velocity of 4π rad/s.
A particle of mass m is placed on the inside rough surface of the cone. The particle remains at rest relative to the cone.

The coefficient of friction between the particle and the cone is $\frac{1}{4}$.
The semi-vertical angle of the cone is $30°$ and the particle is a distance ℓ m from the vertex of the cone.

Find the maximum value of ℓ, correct to two places of decimals.



Question — 2000 Q6 (a).

(a) A particle is placed on a horizontal rotating turntable, 10 cm from the centre of rotation. There is a coefficient of friction of 0.4 between the particle and the turntable. If the speed of the turntable is gradually increased, at what angular speed will the particle begin to slide?